Theoretical analysis of delamination in a viscoelastic multilayered bar built-up at both ends

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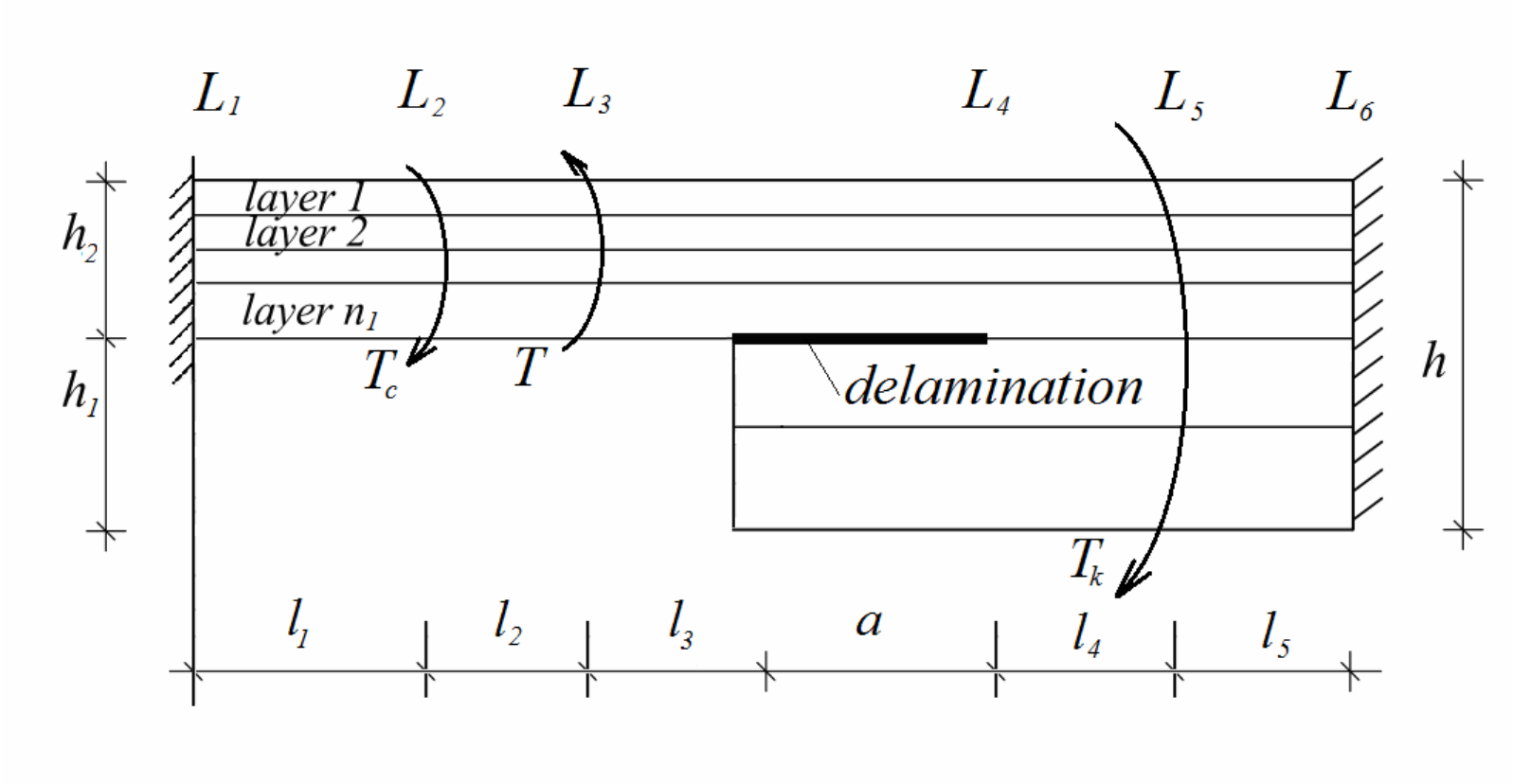
Figure 1. Static model of the bar.



$$\gamma_i = \frac{\tau_i}{\theta_i^2 \eta_{Qi}} \left(\frac{1}{\eta_{Qi}} - \frac{\beta_i}{\theta_i} \right) \left(e^{-\theta_i t} - 1 \right) + \frac{\beta_i \tau_i}{2\theta_i} t + \frac{\tau_i}{\theta_i} \left(\frac{1}{\eta_{Qi}} - \frac{\beta_i}{\theta_i} \right) + \frac{\tau_i}{E_{Ri}}, \qquad (2)$$

$$\theta_i = \frac{E_{Ri}}{\eta_{Qi}} \left(1 + \frac{\eta_{Qi}}{\eta_{Di}} \right), \qquad (3)$$

$$\beta_i = \frac{E_{Ri}}{\eta_{Di} \eta_{Qi}}, \qquad (4)$$

$$\tau_i = \delta_i t_n \qquad (5)$$

$$i = 1, 2, \ldots, n . \qquad (6)$$

$$\varphi_{L1} = 0 ; \tag{7}$$

$$\varphi_{L1} = \frac{I_1}{S} l_1 + \frac{I_2}{S} l_2 + \frac{I_3}{S} (l_3 + a) + \frac{I_4}{S_g} l_4 + \frac{I_5}{S_g} l_5 ; \tag{8}$$

$$\varphi_{L2} = \frac{I_2}{S} l_2 + \frac{I_3}{S} (l_3 + a) + \frac{I_4}{S_g} l_4 + \frac{I_5}{S_g} l_5 ; \tag{9}$$

$$\varphi_{L5} = \frac{I_5}{S_g} l_5 . \tag{10}$$

$$T_1 + T_c + T_2 = 0, \quad (11)$$

$$T_2 + T + T_3 = 0, \quad (12)$$

$$T_3 + T_4 = 0, \quad (13)$$

$$T_4 + T_k + T_5 = 0, \quad (14)$$

$$T_c = c\varphi_{12}, \quad (15)$$

$$T_k = k\varphi_{15}, \quad (16)$$

$$G = \frac{dU}{dA}, \quad (17)$$

$$dA = bda, \tag{18}$$

$$U = U_{1,2} + U_{2,3} + U_{3,4} + U_{4,5} + U_{5,6}. \tag{19}$$

$$U_{2,3} = l_2 \sum_{i=1}^{n_1} \iint_{(A_i)} u_{0,2,3,i} dA_i, \tag{20}$$

$$U_{3,4} = (l_3 + a) \sum_{i=1}^{n_1} \iint_{(A_i)} u_{0,3,4,i} dA_i, \tag{21}$$

$$U_{4,5} = l_4 \sum_{i=1}^{n} \iint_{(A_i)} u_{0,4,5,i} dA_i, \tag{22}$$

$$U_{5,6} = l_5 \sum_{i=1}^{n} \iint_{(A_i)} u_{0,5,6,i} dA_i, \tag{23}$$

$$u_{0i} = \frac{1}{2} \tau_i \gamma_i \tag{24}$$

Figure 2. Schema of the viscoelastic model.

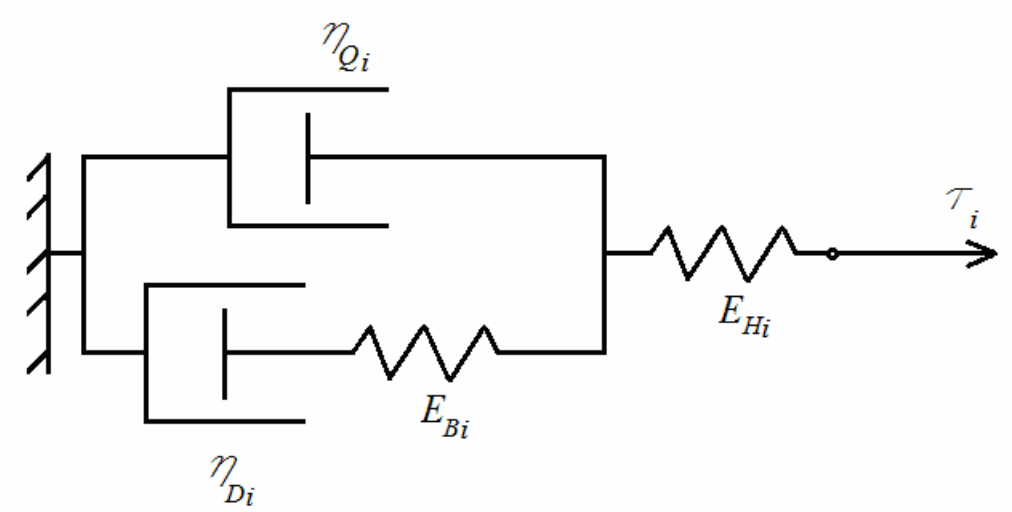


Figure 3. The SERR versus c (curve 1 – for d=0.00001 kNm/s, curve 2 – for d=0.0001 kNm/s and curve 3 – for d=0.001 kNm/s).

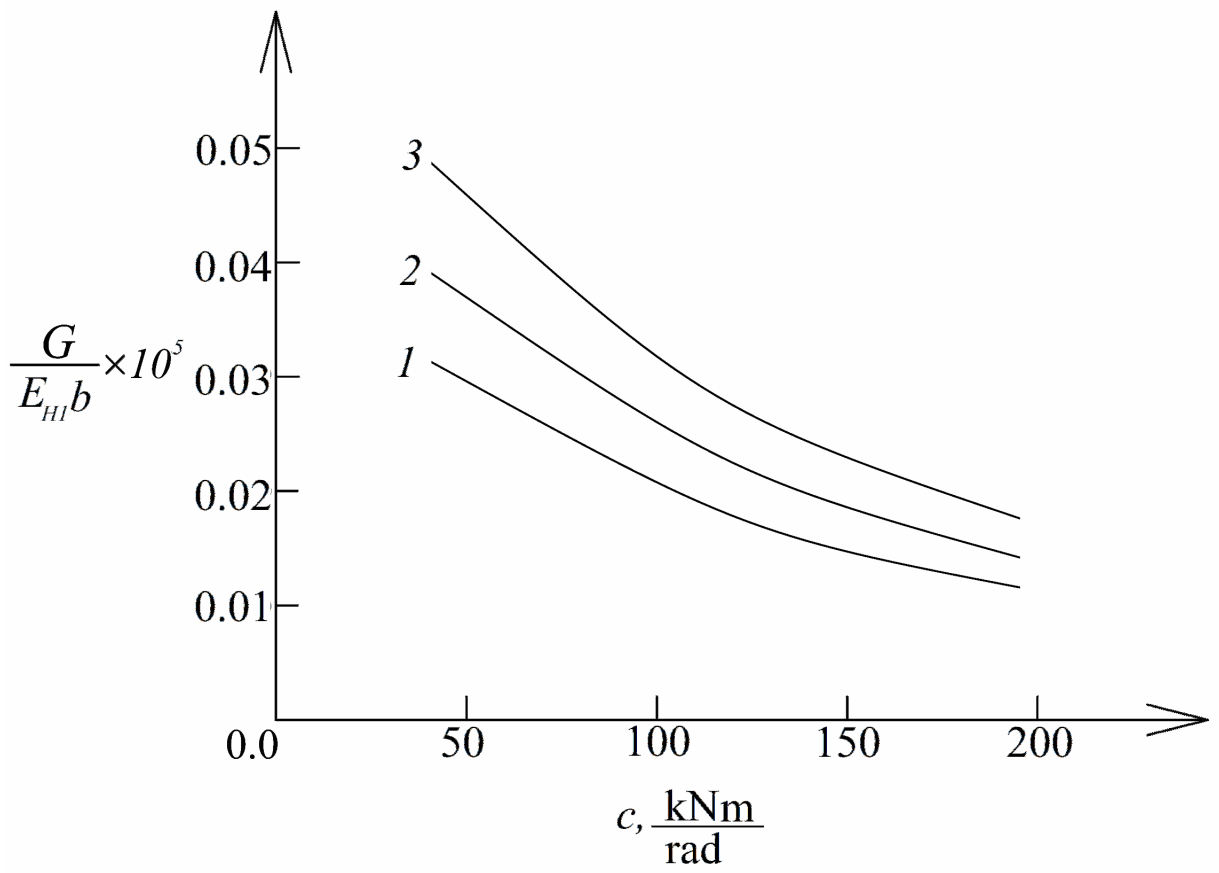


Figure 4. The SERR versus k (curve 1 – for E2/E1=0.5, curve 2 – for E2/E1=1.0 and curve 3 – for E2/E1=2.0).